Optical Navigation Models in ODTK

May, 2023

GOVERNMENT INITIATIVES (AGI)

Overview

- The Orbit Determination Tool Kit (ODTK) implements Optical Navigation (OpNav) measurement models
  - Useful for deep space navigation on approach to target bodies
  - Useful in Cis-Lunar space as backup or primary navigation method
- OpNav measurements are not standardized in terms of how they are generated from raw images
  - The secret sauce tends to be the algorithm for converting raw images to useful measurements of direction and distance
  - ODTK treats this process as a “black box” and only attempts to model common error sources based on literature review

Primary References

- Owen, W., “Methods of Optical Navigation,” AAS/AIAA Space Flight Mechanics Meeting, AAS Paper 2011-215, Washington, DC, Feb 2011.
- Holt, Greg & D'Souza, Christopher & Saley, David, “Orion Optical Navigation Progress Toward Exploration Mission 1”. AAS/AIAA Space Flight Mechanics Meeting, AAS Paper 2018-1978, Kissimmee, FL, Jan 2018.

Basic steps to generate OpNav bearing measurements

- Spacecraft camera is oriented to point to target body
- Image is taken of target body – received as a matrix of pixel values
- Centroid of target body is determined from image
  - Image processing algorithms are an active area of research
  - Result is in pixel coordinates (real numbers)
- Pixel coordinates are transformed to a direction in the camera frame
- Direction in camera frame is transformed to a direction in the inertial frame
  - Reported as Right Ascension / Declination
- For resolved images, distance may be determined based on the size of the target body in the image and a model of target body shape

Each step in the process is subject to error

- Spacecraft camera is oriented to point to target body
  - Subject to error in commanded spacecraft attitude, camera alignment in spacecraft body frame and target body location
  - Does not affect OD processing directly
- Image is taken of target body
  - Subject to pixel based errors, stray light and other environmental factors
  - Does not affect OD directly, but may affect image processing algorithms
- Centroid of target body is determined from image
  - Subject to algorithm specific biases and noise
  - Common bias in image processing moves centroid location in camera-target-Sun plane

Each step in the process is subject to error cont.

- Pixel coordinates converted to direction in the camera frame
  - Subject to errors in ratio of pixel size to focal length
  - Subject to distortion, tip and tilt (real vs perfect optics and image plane orientation)
- Direction in camera frame is transformed to a direction in the inertial frame
  - Subject to errors in spacecraft attitude and camera orientation in spacecraft body frame
- For resolved images, distance may be determined based on model of target body shape
  - Subject to errors in the body shape model, topography along the limb, effects of body atmosphere

Bearing measurements

Right Ascension and Declination of Target Centroid

GOVERNMENT INITIATIVES (AGI)

Starting with the output of the image processing

- Centroid in pixel coordinates (u,v) where u is horizontal and v is vertical
- Transform to "corrected" image coordinates (x', y') in distance units (mm)
  - Corrected coordinates have removed the effects of distortion, tip and tilt

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} K_x^{-1} & K_{xy}^{-1} \\ K_{yx}^{-1} & K_y^{-1} \end{bmatrix} \begin{pmatrix} u & - u_0 \\ v & - v_0 \end{pmatrix}$$

K^{-1}= inverse of K

  - K_x^{-1} is the horizontal detector pitch (pixel size), typically given in mm
  - K_y^{-1} is the vertical detector pitch (pixel size), typically given in mm
  - K_{xy}^{-1} and K_{yx}^{-1} are typically zero
  - u_0 and v_0 are the pixel coordinates of the optical axis

Ansys
GOVERNMENT INITIATIVES (AGI)

Transform to Image Coordinates

- Transform to uncorrected image coordinates

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} - \begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix}$$

  - ϵ_1 = distortion coefficient
  - ϵ_2 = tip
  - ϵ_3 - tilt
  - $r^2 = x^2 + y^2$
- Correction parameters (ϵ_1, ϵ_2, ϵ_3) are nominally zero
  - Could be estimated or provided
  - Ultimately not included in the ODTK implementation

Transform to Camera Coordinates I

- Transform to camera coordinates from image coordinates

$$\begin{pmatrix} \varrho_1^C / \varrho_3^C \\ \varrho_2^C / \varrho_3^C \end{pmatrix} = \frac{1}{f} \begin{pmatrix} x \\ y \end{pmatrix}$$

  - ϱ_1^C = X component of camera to target vector in the camera frame
  - ϱ_2^C = Y component of camera to target vector in the camera frame
  - ϱ_3^C = Z component of camera to target vector in the camera frame
  - f = focal length of camera in same units as image plane coordinates (mm)

Transform to Camera Coordinates II

- Substituting in for x and y we get the expression for direction in camera coordinates as a function of the pixel coordinates. The camera error terms are left expressed in terms of x and y for simplicity

$$\begin{pmatrix} \varrho_1^C/\varrho_3^C \\ \varrho_2^C/\varrho_3^C \end{pmatrix} = \frac{1}{f}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{f}\left[\begin{bmatrix} K_x^{-1} & K_{xy}^{-1} \\ K_{yx}^{-1} & K_y^{-1} \end{bmatrix}\begin{pmatrix} u & -u_0 \\ v & -v_0 \end{pmatrix} - \begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix}\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix}\right]$$

- Assuming K_{xy}^{-1} and K_{yx}^{-1} are zero and defining $S^{-1} = \frac{1}{f} K^{-1},\ S = f\ K$.

$$\begin{pmatrix} \varrho_1^C/\varrho_3^C \\ \varrho_2^C/\varrho_3^C \end{pmatrix} = \begin{bmatrix} S_x^{-1} & 0 \\ 0 & S_y^{-1} \end{bmatrix}\begin{pmatrix} u & -u_0 \\ v & -v_0 \end{pmatrix} - \frac{1}{f}\begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix}\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix}$$

Note: K^{-1} has units of mm, f has units of mm, S is unitless

Transform to Camera Coordinates III

- Concentrating on the first term of the right hand side, we see that the coordinates in the camera frame are simply a translated and scaled version of the coordinates in the image plane in pixel space. We also note that an error in the scale factor(s) will have the effect of moving the observation toward or away from the optical axis.

$$\begin{pmatrix} \varrho_1^C / \varrho_3^C \\ \varrho_2^C / \varrho_3^C \end{pmatrix} \sim \left[\begin{bmatrix} S_x^{-1} & 0 \\ 0 & S_y^{-1} \end{bmatrix} \begin{pmatrix} u & - u_0 \\ v & - v_0 \end{pmatrix} \right]$$

- Noting that the elements on the left are just the tangents of angles to the projections of the relative position vector

$$\begin{pmatrix} \tan \alpha_1 \\ \tan \alpha_2 \end{pmatrix} \sim \left[\begin{bmatrix} S_x^{-1} & 0 \\ 0 & S_y^{-1} \end{bmatrix} \begin{pmatrix} u & - u_0 \\ v & - v_0 \end{pmatrix} \right]$$

Transform to direction in inertial coordinates

- Let the transformation from camera coordinates to inertial coordinates be represented as:

$$\varrho^{I} = M_{B}^{I}\, M_{C}^{B}\, \varrho^{C}$$

  - Where M_{B}^{I} is the spacecraft body to inertial transformation matrix and M_{C}^{B} is the camera to spacecraft body transformation matrix. M_{B}^{I} is time dependent and subject to spacecraft attitude errors while M_{C}^{B} is constant and subject to a constant alignment error.

Bias and noise sources

***ODTK setting name**

- Biases
  - UVBias* - In pixel space (b_u, b_v)
  - OpticalScaleBias* - Uncertain ratio of pixel size to focal length (b_{S_x})
  - ImageProcessingBias* - Resulting from image processing (b_{IP})
- Noise
  - PixelSpaceDirectionalNoise* - In pixel space (σ_u, σ_v)
  - Resulting from image processing, also in pixel space
    - No separate implementation in ODTK
- Attitude and camera alignment errors
  - Attitude errors can be represented as a bias matrix $b_{M_B^I}$
  - Camera alignment errors can be represented as a bias matrix $b_{M_C^B}$
  - No current model in ODTK

Computed measurement conceptual build up

Process

- Camera takes picture of target body
- Image processing applied to determine body centroid
- Centroid mapped to inertial observation

Model

- Compute the centroid location in pixel coordinates as the camera would see it (including pointing and optical biases)
- Add biases due to image processing
- Construct inertial observations using OpNav model for biases and corrections

Computed measurement conceptual build up

- Compute apparent location of target body relative to camera
- Determine camera attitude
- Map apparent relative location into camera coordinates using attitude and camera alignment corrections
- Map into pixel coordinates using corrected pixel pitch to focal length ratio
- Apply any image processing related biases (measurement production modeled)
- Map back to camera coordinates using uncorrected transforms
- Map back to inertial coordinates ignoring attitude and camera mis-alignment

Computed bearing measurement buildup with biases

$$\varrho^I = (R_{Target} - R_C)/\|R_{Target} - R_C\|$$

Apparent Position

$$\varrho^B = \left[b_{M_I^B}\right][M_I^B]\varrho^I$$

Body Coordinates

$$\varrho^C = \left[b_{M_B^C}\right][M_B^C]\varrho^B$$

Camera Coordinates

$$\binom{x}{y} = f\begin{pmatrix} \varrho_1^C/\varrho_3^C \\ \varrho_2^C/\varrho_3^C \end{pmatrix}$$

Image Coordinates

$$\binom{x'}{y'} = \binom{x}{y} + \begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix}\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix}$$

Corrected Image Coordinates

Bearing measurement build up II

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} K_x + b_{K_x} & K_{xy} \\ K_{yx} & K_y + b_{K_y} \end{bmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} b_u \\ b_v \end{pmatrix}$$

Pixel Coordinates

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} S_x + b_{S_x} & S_{xy} \\ S_{yx} & S_y + b_{S_y} \end{bmatrix} \frac{1}{f} \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} b_u \\ b_v \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} S_x + b_{S_x} & S_{xy} \\ S_{yx} & S_y + b_{S_y} \end{bmatrix} \frac{1}{f} \left[\begin{pmatrix} x \\ y \end{pmatrix} + \begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix} \right] + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} b_u \\ b_v \end{pmatrix}$$

Bearing measurement build up III

Pixel Coordinates

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} S_x + b_{S_x} & S_{xy} \\ S_{yx} & S_y + b_{S_y} \end{bmatrix} \begin{pmatrix} \varrho_1^C / \varrho_3^C \\ \varrho_2^C / \varrho_3^C \end{pmatrix} +$$

$$\frac{1}{f} \begin{bmatrix} S_x + b_{S_x} & S_{xy} \\ S_{yx} & S_y + b_{S_y} \end{bmatrix} \begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} b_u \\ b_v \end{pmatrix}$$

Image Processing

Finally, there is bias introduced by the image processing

$$\begin{pmatrix} u \\ v \end{pmatrix}_{IP} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} b_u \\ b_v \end{pmatrix} + \begin{bmatrix} S_x + b_{S_x} & S_{xy} \\ S_{yx} & S_y + b_{S_y} \end{bmatrix} \begin{pmatrix} \varrho_1^C / \varrho_3^C \\ \varrho_2^C / \varrho_3^C \end{pmatrix} +$$

$$\frac{1}{f} \begin{bmatrix} S_x + b_{S_x} & S_{xy} \\ S_{yx} & S_y + b_{S_y} \end{bmatrix} \begin{bmatrix} xr^2 & xy & x^2 \\ yr^2 & y^2 & xy \end{bmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_2 \end{pmatrix} + \begin{pmatrix} \Delta u_{IP}(b_{IP}, \varphi) \\ \Delta v_{IP}(b_{IP}, \varphi) \end{pmatrix}$$

where φ is the solar phase angle

ODTK Implementation

- ODTK model assumes that
  - Optical correction parameters (ϵ_1, ϵ_2, ϵ_3) can be ignored (very small or accounted for in image processing)
  - The combination of attitude and camera alignment errors can be accommodated through noise and other bias states
  - Pixels are rectangles or squares

$$\begin{pmatrix} u \\ v \end{pmatrix}_{IP} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{bmatrix} S_x + \boldsymbol{b_{S_x}} & 0 \\ 0 & S_y + \boldsymbol{b_{S_y}} \end{bmatrix} \begin{pmatrix} \varrho_1^C / \varrho_3^C \\ \varrho_2^C / \varrho_3^C \end{pmatrix} + \begin{pmatrix} \Delta u_{IP}(\boldsymbol{b_{IP}}, \varphi) \\ \Delta v_{IP}(\boldsymbol{b_{IP}}, \varphi) \end{pmatrix} + \begin{pmatrix} \boldsymbol{b_u} \\ \boldsymbol{b_v} \end{pmatrix}$$

Optical scale bias assumed to same in X and Y directions: $\boldsymbol{b_{S_y}} = \boldsymbol{b_{S_x}}$

Ansys GOVERNMENT INITIATIVES (AGI)

A simple image processing bias model

- Based on "Methods of Optical Navigation"
  - Bias is in Camera-Target-Sun plane, positive in direction of Sun
  - Bias is related to sun phase angle, φ, as a power of sin(φ/2)
    - The exponent (n) of the sine function is user configurable
  - D is the diameter of the target body

$$\Delta u_{IP}(\boldsymbol{b_{IP}}, \varphi) = C_u\, \boldsymbol{b_{IP}}\, \frac{D}{T}\, sin^n\left(\frac{\varphi}{2}\right)$$

$$\Delta v_{IP}(\boldsymbol{b_{IP}}, \varphi) = C_v\, \boldsymbol{b_{IP}}\, \frac{D}{T}\, sin^n\left(\frac{\varphi}{2}\right)$$

$$T = \left\| R_{Target} - R_C \right\|$$

Ansys GOVERNMENT INITIATIVES (AGI)

Point-type bearing measurements

- If the target body is not resolved in the image, measurements should be input as “Point” measurements
  - The image processing bias is not applied
- If the ODTK PointObservationMethod is set to Astrometric, then the right ascension and declination measurements are assumed to have been determined by comparison to a star field instead of being reduced from camera coordinates.
  - U/V biases are not applied
  - The optical scale bias is not applied
  - The image processing bias is not applied
  - Measurement noise taken from RA/Dec measurement statistics

Range measurements

Distance to Target Center

GOVERNMENT INITIATIVES (AGI)

Range measurement model summary

- Observed range will be formed using one or more measurements of the target body size on the image plane of the camera
  - Size is originally measured in pixel space
  - Size is converted to a distance measurement using the modeled shape of the target body, the detector pitch and the focal length
- Computed range will be calculated using the current estimates of the camera and target body center locations
- A bias in the target size will be estimated. The observed range model provides the relationship between target size and range.
- Range measurement noise will be expressed as a function of surface roughness and we will use the relationship between target size and range to map the noise from target space to range space

Observed range model

- The observed model for range refers to how the range measurement is constructed. Reference Figure 3 on page 5 of the paper on Orion Optical Navigation Progress.

$$\rho_{obs} = R_T \sqrt{1 + \left(\frac{2KF}{n_d}\right)^2}$$

  - R_T is the radius of the target body
  - K is the inverse detector pitch in pixels/mm
  - F is the focal length in mm
  - n_d is the target diameter in pixels measured in the image plane
  - Note that n_d is the actual measured quantity, but there could be more than one n_d type measurement being used to construct a single reported range measurement

Target scale bias model

- Reported range is directly proportional to the modeled size of the target body.
- Errors in the shape model for the target, will result in errors in the reported range.
- The target scale bias $\frac{\Delta R_T}{R_T}$ accommodates a systematic bias in the size (too large or too small) of the target body.

$$\rho_{obs} = R_T\left(1 + \frac{\Delta R_T}{R_T}\right)\sqrt{1 + \left(\frac{2KF}{n_d}\right)^2}$$

Ansys
GOVERNMENT INITIATIVES (AGI)

Computed range model

- The computed range model provides the implementation for inside the OD software. In this case, we will replace the true range part of the computation with the distance between the camera and the center of the target body. We will also now use a bias notation for the error in the target body size.

$$\rho_{true} = \left\| R_{Target} - R_C \right\|$$

$$\rho_{comp} = \left(1 + b_{\Delta R_T Rel}\right) \rho_{true}$$

$$\frac{\partial \rho}{\partial b_{\Delta R_T Rel}} = \rho_{true}$$

$$b_{\Delta R_T Rel} = \frac{\Delta R_T}{R_T}$$

Range noise

- Noise in the range model is assumed to originate from roughness in the surface plus noise associated with the construction of range measurement from the image.
- A surface deviation along the limb, ΔR_{T_rough}, can affect the value of the range measurement.

$$\rho_{obs} = \left(R_T + \Delta R_{T\_rough}\right)\sqrt{1 + \left(\frac{2KF}{n_d}\right)^2}$$

Roughness mapped into range noise

$$\sigma_\rho^2 = \frac{\partial \rho_{obs}}{\partial \Delta R_T} \sigma_{\Delta R_{T\_rough}}^2 \frac{\partial \rho_{obs}}{\partial \Delta R_T}$$

ODTK OpNav Error Model

Bias and noise

Ansys

GOVERNMENT INITIATIVES (AGI)

ODTK Optical Navigation Bias States

- Bias states in pixel coordinates ($\boldsymbol{b_u}$, $\boldsymbol{b_v}$)
  - Applies to Point and Limb based directional measurements
- Correction to ratio of pixel pitch to focal length ($\boldsymbol{b_{S_x}}$, $\boldsymbol{b_{S_y}}$)
  - ODTK assumes pixels are square, only one state needed ($\boldsymbol{b_{S_y}} = \boldsymbol{b_{S_x}}$)
  - Applies to Point and Limb based directional measurements
- Image processing bias state ($\boldsymbol{b_{IP}}$)
  - Implemented in camera-target-Sun plane – centroid biased towards Sun
  - Applies to Limb-based directional measurements
- Target body scale factor ($\boldsymbol{b}_{\Delta R_T Rel}$)
  - Applies to Range measurements

ODTK Optical Navigation measurement noise inputs

- Measurement noise in pixel coordinates ($\boldsymbol{\sigma_u}$, $\boldsymbol{\sigma_v}$)
  - Assumed to be the same in both directions ($\boldsymbol{\sigma_v} = \boldsymbol{\sigma_u}$)
  - Applies to Limb and Point, non-astrometric bearing measurements
- Measurement noise in RA/Dec coordinates
  - Applies to Point, astrometric bearing measurements
- Noise accounting for topography along the limb
  - Applies to range measurements